

On Hypothesis of the Two Large Extradimensions

Janusz Garecki

Institute of Physics, University of Szczecin, Wielkopolska 15; 70-451 Szczecin, POLAND

(February 7, 2008)

Abstract

Recently there was proposed a hypothesis about existence of the two large extradimensions. This hypothesis demands, e.g., modification of Newton law at submillimeter scale. In this brief report we show that this hypothesis cannot be correct in present formulation.

KEY WORDS: dimensionality of the spacetime, spacetime structure.

04.20.Cv

I. THE HYPOTHESIS, ITS CONSEQUENCES AND CONCLUSION

In order to solve the so-called “hierarchy problem” and give an explanation why gravity is so weak in comparison with other known interactions some authors [1–5] proposed recently to use *large extradimensions*. Namely, these authors assert that the gravity is really strong and the *electroweak unification energy scale* $M_u = 1 \text{ Tev}$ ($= 10^{19}m^{-1}$ in natural units in which $\hbar = c = 1$) and the fundamental Planck’s scale $M_P = 10^{16} \text{ Tev}$ ($= 10^{35}m^{-1}$) *are indeed the same size*, but four-dimensional gravity is so weak (hence M_P is so large) due to dilution of gravity (Why only gravity?) in large extradimensions. So, these authors assert that the unification energy $M_u = 1 \text{ Tev}$ *is the only fundamental scale* in Nature. If so, new dimensions, black holes, quantum gravity, and string theory will become experimentally accessible in near future [6–8].

Following [1–4] we have the following formula on radii R_c of compactified extradimensions

$$R_c = \frac{\hbar \times c}{M_u} \left(\frac{M_P}{M_u} \right)^{\frac{2}{n}}, \quad (1)$$

where n denotes number of compactified extradimensions. $\frac{M_P}{M_u} = 10^{16}$ if $M_u = 1 \text{ Tev}$.

From (1) one can easily get that if:

1. $n = 1$, then $R_c \approx 1AU$,
2. $n = 2$, then $R_c = 10^{-3}m$,
3. $n = 7$, then $R_c = 10^{-14}m$.

The most popular is the second case in which two spatial extradimensions are curled up into circles about 10^{-3} m in size. In this case we have five-dimensional space and six-dimensional spacetime.

The first possibility must be rejected at once because it is from the beginning *incompatible with experience*.

As we will show the second possibility *also should be rejected* at least from the two following causes.

Firstly, it was shown in past by P. Ehrenfest, G.J. Whitrow and others [9–11] that the three-dimensional space, i.e., four-dimensional spacetime *is necessary for many reasons*, e.g., only in a three-dimensional space atoms can be stable. So, if hypothesis about two large extradimensions which have radii $R_c = 10^{-3}m \gg 10^{-10}m$ ($10^{-10}m$ is a typical diameter of an atom) were correct than *our existence would be impossible*.

Secondly, if the hypothesis on two large spatial extradimensions was correct then the Newton law *had to be changed* in the scale $r \leq R_c = 10^{-3}$ m. It can be most easily seen by use of the *Gauss law* in N – dimensional space for a point mass m

$$\int_{x_1^2+x_2^2+\dots+x_n^2=R^2} \vec{E} \cdot \vec{n} d\sigma = m. \quad (2)$$

Here \vec{E} means the gravitational strength, \vec{n} is the unit normal to the imagined Gauss sphere and $d\sigma$ denotes an integration element over this sphere.

Using spherical symmetry one can easily obtain from (2)

$$E(R) \int_{x_1^2+x_2^2+\dots+x_n^2=R^2} d\sigma = E(R) \frac{N\pi^{N/2}R^{N-1}}{\Gamma(N/2+1)} = m. \quad (3)$$

From (3) there follow

$$E(r) = \frac{G_N m}{r^{N-1}}, \quad (4)$$

and *the modified Newton law* for the value of the gravitational force F between two point masses m_1, m_2 if $r \leq R_c = 10^{-3}$ m

$$F = \frac{G_N m_1 m_2}{r^{N-1}}. \quad (5)$$

Here G_N denotes a new gravitational constant. We have [1–4]

$$G_N = (M_u)^{-(2+n)}, \quad M_u = 1TeV (= 10^{19}m^{-1}), \quad (6)$$

where $n = N - 3$ is the number of the curled up spatial extradimensions.

For $N = 5$, i.e., for $n = 2$, we get from (5-6)

$$F = \frac{G_N m_1 m_2}{r^4}, \quad G_N = (M_u)^{-4} = 10^{-76} m^4. \quad (7)$$

In the following we will confine ourselves to the last, most popular possibility when $n = 2$, i.e., we confine to the five-dimensional space and to the six-dimensional spacetime.

We will show that the modification Newton law for $r \leq R_c = 10^{-3}$ m given in the case by formula (7) *cannot be correct*.

With this aim we will use an old Stanford experiment [12–13] on free falling conductivity electrons inside of a freely standing or freely hanging metal (inside Cu). This experiment, performed with very high precise, showed that the conductivity electrons in such a metal (Cu) were falling (under influence of the Earth gravitational field) *with the same acceleration g_{int} inside metal (Cu) as in vacuum*, i.e., they showed that $g_{int} = g_{ext} = 9,8 \frac{m}{s^2}$.

Such result is okay if we apply *the same, ordinary* Newton law to gravitational interaction between an electron inside of the metal and Earth and to gravitational interaction of this electron and a positive ion of the cristal lattice of the metal (Cu). Namely, the simple calculation shows that for *ordinary Newton law* the ratio of the values of the gravitational forces between electron-Earth (F_{e-E}) and between electron-ion (F_{e-ion}) is equal

$$\frac{F_{e-E}}{F_{e-ion}} \approx 0.3 \times 10^{16}. \quad (8)$$

Calculating the ratio (8) we have taken $m_{ion} = 108 \times 10^{-27} kg$, $M_{Earth} =: M_E = 6 \times 10^{24} kg$, $R_{Earth} =: R_E = 6,4 \times 10^6 m$ (R_E = distance between an electron inside of the metal and the center of the Earth) and $r = 0,5 \times 10^{-10} m$ as an upper value of the distance between a conductivity electron and the nearest positive ion inside of the metal (of course the distance between conductivity electrons and positive ions inside of the metal can be smaller than the last value).

The result (8) shows that the ordinary Newton gravitational interaction between conductivity electrons and positive ions of the metal *is negligible* in comparison with ordinary Newton's gravitational interaction between the same electrons and Earth. In consequence, the conductivity electrons inside of a metal can freely fall with the same acceleration as in

vacuum, and give an uniform, gravity induced electrostatic field inside of the metal [12,14]. This gravity induced electrostatic field is in an equilibrium with the gravitational field which acts between an electron inside a metal and Earth.

However, if the hypothesis about the two large extradimensions curled up to the size $R_c = 10^{-3}$ m is correct, then *one should change Newton law* for gravitational interaction between a conductivity electron and a positive ion of the metal cristal lattice to the form (7), i.e., to the form

$$F_{e-ion} = \frac{G_N m_e \times m_{ion}}{r^4}, \quad G_N = (M_u)^{-4} = 10^{-76} m^4, \quad (9)$$

because an upper limit of the distance distance, r , between interacting particles is in the case $r \approx 0,5 \times 10^{-10} m \ll 10^{-3} m = R_c$ (and, of course, the distances between conductivity electrons and positive ions in general can be smaller). On the other hand, the Newton law for gravitational interaction between a conductivity electron and Earth *should be unchanged*, i.e., it should have the ordinary form

$$F_{e-E} = \frac{G m_e \times M_E}{R_E^2}, \quad G = (M_{Pl})^{-2} = 10^{-70} m^2, \quad (10)$$

because in this case the distance between interacting bodies (electron–Earth) is of order $R_E = 6,4 \times 10^6$ m $\gg R_c = 10^{-3}$ m.

After doing so one can easily calculate that then, the ratio of the values of the gravitational forces

$$\frac{F_{e-ion}}{F_{e-E}} \quad (11)$$

is greater than 1 already for $r \leq 3 \times 10^{-11}$ m, e.g., if $r = 3 \times 10^{-11}$ m, then this ratio is $\approx 74 > 1$. Here r means a distance between an conductivity electron and an ion of the cristal lattice of the metal.

But in such situation the conductivity electrons *could not freely fall* inside a metal in the same way as they do in vacuum, i.e., with the same uniform acceleration as they freely fall in vacuum. The conductivity electrons inside of a metal *should fall with an effective*

acceleration $g_{eff} = g_{int} \neq g_{ext} = 9,8 \frac{m}{s^2}$ and they rather should fall “onto” positive ions of the cristal lattice instead of onto Earth. Especially in a near vicinity of the metal walls and when if there would be cristal lattice defects inside of the metal.

Thus, we conclude from this old Stanford experiment that *one cannot change Newton law at least up to distances $r \approx 10^{-11}$ m* in the manner expected by the hypothesis about two large extradimensions¹. The radii R_c of the curled up into circles two spatial extradimensions, if they really exist, *should be smaller* than $R_c = 10^{-11}$ m , i.e., they should be much smaller than recently proposed $R_c = 10^{-3}$ m [1–8], and, in consequence, the unification energy scale *should be much greater* than $M_u = 1$ Tev.

It is interesting in this context that the recent experiments [14,15] discovered *no deviations* from Newton’s law up to distances $r \approx 0,2mm$.

Of course, one can preserve $M_u = 1$ Tev as an admissible unification energy scale but this demand *increasing* of the number of the compactified extradimensions to seven,i.e., this demand eleven-dimensional spacetime. Then the radii R_c of these seven extradimensions should be curled up, as we have mentioned about that already, to the size $R_c \approx 10^{-14}$ m.

Summing up, we think that the hypothesis about two large spatial extradimensions with $R_c \approx 10^{-3}$ m and about $M_u = 1$ Tev as the grand unification energy scale *is incorrect*. Our existence and experiments *contradict this hypothesis*.

If the extradimensions really exist (this is very problematic and controversial, see eg., [16]), then they should be compactified in a much smaller scale than the proposed scale $R_c \approx 10^{-3}$ m, e.g., in the scale $R_c \leq 10^{-14}$ m. The most probably they should be compactified in the Planck’s scale, i.e., in the scale originally proposed. But then we return back to the “hierarchy problem” and to the cosmological constant problem [16].

¹Of course there are always possible modifications which follow from general relativity

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